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LUMPS AND P-BRANES IN OPEN STRING FIELD THEORY

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ABSTRACT

We describe numerical methods for constructing lump solutions in open string field theory. According to Sen, these lumps represent lower dimensional Dp-Branes and numerical evaluation of their energy can be compared with the expected value for the tension. We take particular care of all higher derivative terms inherent in Witten's version of open string field theory. The importance of these terms for off shell phenomena is argued in the text. Detailed numerical calculations done for the case of general p brane show very good agreement with Sen's conjectured value. This gives credence to the conjecture itself and establishes further the usefulness of Witten's version of SFT .

1 Introduction

Since its introduction, string field theory held out the promise for nonperturbative studies of string theory. Recently Sen [1, 2] has argued that open bosonic (string) field theory describes the dynamics of $D\bar{D}$ system with the tachyon providing the instability inherent in such pair. Tachyon condensation also describes the decay of a single unstable D-brane. In addition, according to [2] the kink and lump solutions of such field theory lead to lower dimensional branes. In recent work Sen and Zweibach [3] studied in detail tachyon condensation in 26D open string theory. This follows the earlier, pioneering work of Samuel and Kostelecky who were the first to consider the vacuum structure of string field theory [4, 5]. They used a level truncation scheme to generate an approximation for the tachyon effective potential. Following this scheme, results were found [3] that show great agreement in numerical values with the expected exact results. Similar results were recently obtained for superstring field theory [6, 7]. Impressive high level studies appeared in [8, 9].

It is equally important to give a construction of non-constant kink and lump-like solutions. One can expect that it is for these that the stringy effects present in string field theory might play the most important role. One of the characteristic features present in construction of Witten's version of the theory [10, 11, 12, 13] is the appearance in the interaction of terms exponential in derivatives. These terms can be moved from the interaction to give a nontrivial kinetic term. They have frustrated early attempts for construction of nonperturbative soliton (or instanton) solutions of the theory.

In the present work, we consider this problem in numerical terms. Concentrating on the lump of open string theory, we develop methods for its numerical solution. In this we keep the nontrivial exponential terms characteristic of the string vertex interaction and simultaneously perform a level truncation. This, we argue is to provide a very good approximation to the exact result.

The content of the paper is as follows. After a short description of open SFT, we discuss some features relevant to the present work. We explain (based on earlier observations [14]) how and why the approach of keeping higher derivatives and simultaneous level truncation holds the promise for a good approximation. We then proceed to the numerical work.

While this work was in progress, there appeared the work of Ref.[15] which considers the problem in its field theory limit.

2 Open String Field Theory

We begin by describing some features of open string field theory which are of relevance to the investigation that follows. One has the cubic action:

$$S = \langle A|Q|A\rangle + \frac{g}{3}\langle V_3||A\rangle|A\rangle|A\rangle \tag{1}$$

with Q being the first quantized BRST operator. The geometric, three string interaction is realized in the Hilbert space by the vertex

$$\langle V_3| = \langle 0|\exp\{\sum {\alpha_n}^r N^{rs}_{nm}\alpha^s_m + \alpha' \ln\gamma \sum_{r=1}^{3}\partial^2_r\} \tag{2}$$

Here the Neumann coefficient N^{rs} are determined in terms of appropriate conformal mapping ,their explicit values are determined in [10, 11].

One of the main properties of the three-string vertex is an explicit appearance of higher derivative terms. They come in exponential form acting on each string field

$$|A\rangle \rightarrow \exp\left(\alpha' \ln\gamma\partial_\mu\partial^\mu\right)|A\rangle \tag{3}$$

with the constant

$$\gamma = \frac{3\sqrt{3}}{4} \tag{4}$$

The exponential terms are not relevant in studies of vacuum structure but they can have a nontrivial effect in any other nonzero momentum process. Concerning a systematic approximation or expansion scheme one notes the following. The masses of tachyon (and higher mass fields) are proportional to $1/\alpha'$. In general α' serves as a scale ,it can be scaled out in front of the SFT action. At a nonperturbative level, there is no small free parameter and no systematic expansion. Since one is not able(at present) to solve the theory in exact terms, one relies on seemingly and hoc approximations. Such is the process of level truncation. In order to understand more clearly the procedure involved and the relative relevance of particular terms, let us recall the original argument given for the level truncation in an unpublished work of ref.[14]. Considering an approximate calculation of a nonzero momentum amplitude, for example for four tachyons one starts from:

$$A_s = \int_0^1 dx\, x^{-s/2-2} \langle V_{34}(\bar{3}')|b_0 x^R|V_{12}(3')\rangle, \tag{5}$$

representing the s-chanel Feynman diagram.

Using the fact that $x^R \alpha_n x^{-R} = x^n \alpha_n$, as well as similar results for $b's$ and $c's$, we have

$$A_s = \int_0^1 dx\, x^{-s/2-2} \langle V_{34}(\dot{3}) b_0 | V_{12}(\dot{3})\rangle. \tag{6}$$

The dot on V indicates that a_{-n}, b_{-n}, c_{-n} have been replaced by x^n $\alpha_{-n}, x^n b_{-n}, x^n c_{-n}$. Expanding in levels corresponds to expanding in powers of $x = e^{-r}$. With the use the appropriate Neumann coefficients after a straightforward algebra one has

$$A_s = \int_0^1 dx\, x^{-s/2-2} \left(1 - \frac{11^2}{3^6} x^2 + \cdots\right) e^{E(x)} \,. \tag{7}$$

The term in the exponent has the expansion

$$\begin{aligned} E(x) = -\frac{s}{2}\ln\gamma \quad &- \quad \left(\frac{s}{2}+2\right)\left(-\frac{2^3}{3^3}x + \frac{2^2\cdot 19}{3^6}x^2 + \cdots\right) - \\ &- \quad \left(\frac{i}{2}+2\right)\left(-\frac{2^4}{3^3}x - \frac{2^6\cdot 7^2}{3^{10}}x^3 + \cdots\right) - \\ &- \quad \left(\frac{2^4}{3^3}x + \frac{2^3}{3^6}x^2 + \cdots\right) + \left(\frac{26\cdot 5^2}{2\cdot 3^6}x^2 + \cdots\right) \,. \end{aligned} \tag{8}$$

This result can easily be rearranged into a form

$$A_s = \int_0^{z(1)} dz\, z^{-s/2-2}(1-z)^{-t/2-2} \tag{9}$$

where

$$z(x) = \frac{1}{\gamma^2}\, x \left(1 - \frac{2^3}{3^3}x + \frac{2^2}{3^3}\, x^2 + \ldots\right) \,. \tag{10}$$

For agreement with the exact result one would need to have z(1)=0.5 (the s and t -chanel diagrams are to cover the full range (0,1).To the present order in the level expansion we have $z(1) \approx 0.50480$ which is indeed very close to the exact value. A more significant observation is the fact that the main effect is contained in the γ^{-2} factor present in the above expression . That term itself gives $z(1) \approx 0.6$. If we look up the origin of this factor, we see that it comes directly from the exponential higher derivative operator present in the vertex: since the intermediate states in the four point amplitude calculation are not on shell the exponential terms contribute giving the corresponding s dependence.

In this nonzero momentum example, we conclude that it is advantageous to keep the higher derivatives exactly and that this followed by a level truncation is likely to give a good overall approximation. Naturaly one still expects this expansion to only be good for certain range of momenta.

Consider then the string field theory with the tachyon (level 0), but with the higher derivative terms kept exactly. The action evaluated originally in [11] reads

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu T\right)^2 + \frac{1}{2\alpha'}T^2 - \frac{g}{3}\gamma^3\tilde{T}^3 \text{ with } \tilde{T} = e^{\ln\gamma\partial^2}T \tag{11}$$

The first sign that the presence of the exponential term in the cubic interaction profoundly influences the nature of the problem is seen in attempting to evaluate the asymptotics of a possible static solution. In ordinary field theories, kink and lump solution can be asymptoticaly characterized as

$$\phi(x) \sim \phi_0 + a_1 e^{-mx} \tag{12}$$

where ϕ_0 is the constant vacuum solution and m is the physical mass of the scalar field. Based on this, one can write a systematic expansion for the lump

$$\phi(x) = \sum_{m=0}^{\infty} a_n\, e^{-nmx} \tag{13}$$

with the classical equations providing a recursion formula for coefficients a_n. The exponential decay e^{-nmx} has a physical meaning, the lump form factor receives a contribution from n mesons. In attempting an analogue expansion in the case of string field theory, one meets a surprise. After moving the exponential into the kinetic term or equivalently denoting $\tilde{T} = \phi$ the string theory tachyon field equation reads

$$\left(\left(\partial_x^2 + 1\right) e^{-c\partial_x^2} - 2\right) \phi(x) = \bar{g}\phi(x)^2 \tag{14}$$

with $c = 2\ln\gamma = \ln 3^3/4^2$. The Ansatz

$$\phi(x) \sim \phi_0 + a, e^{-\bar{m}x} \tag{15}$$

leads to an eigenvalue equation

$$(\bar{m}^2 + 1)e^{-c\bar{m}^2} - 2 = 0 \tag{16}$$

for $\bar{m}$. This equation turns out to have no real solution. With the exponential present the nature of the lump solution has changed from that of ordinary field theory. In particular the decay at asymtopic infinity in string field theory has to be stronger than a simple exponential. The non-exponential decay of the full lump (kink) solution signals that the form factor will have a more complex physical meaning. This feature also necessitates a purely numerical approach to the problem which we attempt in the next section.

3 Calculations and Results

The prescence of the higher derivative terms in (14) prevent a numerical analysis directly in x space. Upon transforming to momentum space, one finds the following nonlinear integral equation

$$\left(\left(1 - \vec{k}^2\right) e^{c\vec{k}^2} - 2\right) \phi(\vec{k}) = \bar{g} \int d^{25-p}q\phi(\vec{q})\phi(\vec{k} - \vec{q}), \tag{17}$$

where $\vec{k}$ is a $25 - p$ dimensional vector. To solve an integral equation of this type, one typically discretizes the problem and solves the resulting matrix equation. In the case on hand, the resulting matrix equation is nonlinear and one is forced to search for the solution $\phi(\vec{k})$ using a numerical minimization. It is computationally difficult and expensive to minimize functions with a large number of variables, and for this reason we have worked directly with a spherically symmetric ansatz for the tachyon field. Stable numerical solutions were obtained using a lattice having 81 points. This implies a nonlinear minimization problem with 81 parameters. The choice of a good objective function to be used in the minimization as well as an accurate initial guess are crucial to obtain a nontrivial solution. Indeed, in practice, we find that the majority of initial guesses lead to the trivial $\phi = 0$ solution. Our objective function was constructed as follows: Start by making an initial guess, $\hat{G}(k_n)$ for the right hand side of (17). This initial guess is used to compute a value for the tachyon wave function

$$\hat{\phi}(k_n) = \frac{\bar{g}\hat{G}(k_n)}{((1-k_n^2)\,e^{ck_n^2} - 2)}. \tag{18}$$

The wave function $\hat{\phi}(k_n)$ can now be used to compute the value of the left hand side of (17). However, this involves a convolution which must be performed with care. Evaluating the convolution directly in momentum space is not optimal: by exploiting the spherical symmetry of the wave function, the convolution can reduced to the integration over a single angle and a radius. The integrand contains the factor $\hat{\phi}(p_n)\hat{\phi}(\delta_n)$ with

$$\delta_n = |\vec{k} - \vec{p}| = \sqrt{p_n^2 + k_m^2 - 2k_m p_n \cos(\theta_l)}. \tag{19}$$

In general, δ_n does not correpsond to a lattice point and one is forced to interpolate from the known points to find $\hat{\phi}(\delta_n)$. A much more efficient way of performing the convolution is by transforming to x space, squaring the function and then transforming back to k space. By using the spherical symmetry of wave function, the Fourier transform can be reduced to a single integration. Thus, the previous double integration has been replaced by two single integrations, which is more efficient. In addition, the integrations only require a knowledge of $\hat{\phi}$ at the lattice points. After convolving $\hat{\phi}$ with itself to obtain $G(k_n)$, the error, $\mathcal{E}$ to be minimized is constructed as

$$\mathcal{E} = \sum_n |G(k_n) - \hat{G}(k_n)|. \tag{20}$$

The minimization was performed using the `fmins.m` subroutine of MATLAB, which employs a simplex search method. A suitable initial guess is obtained by taking a function which initially falls off slightly faster than a Gaussian, but reaches zero at some finite momentum. As an example we have shown the wavefunction for the D20 brane below.

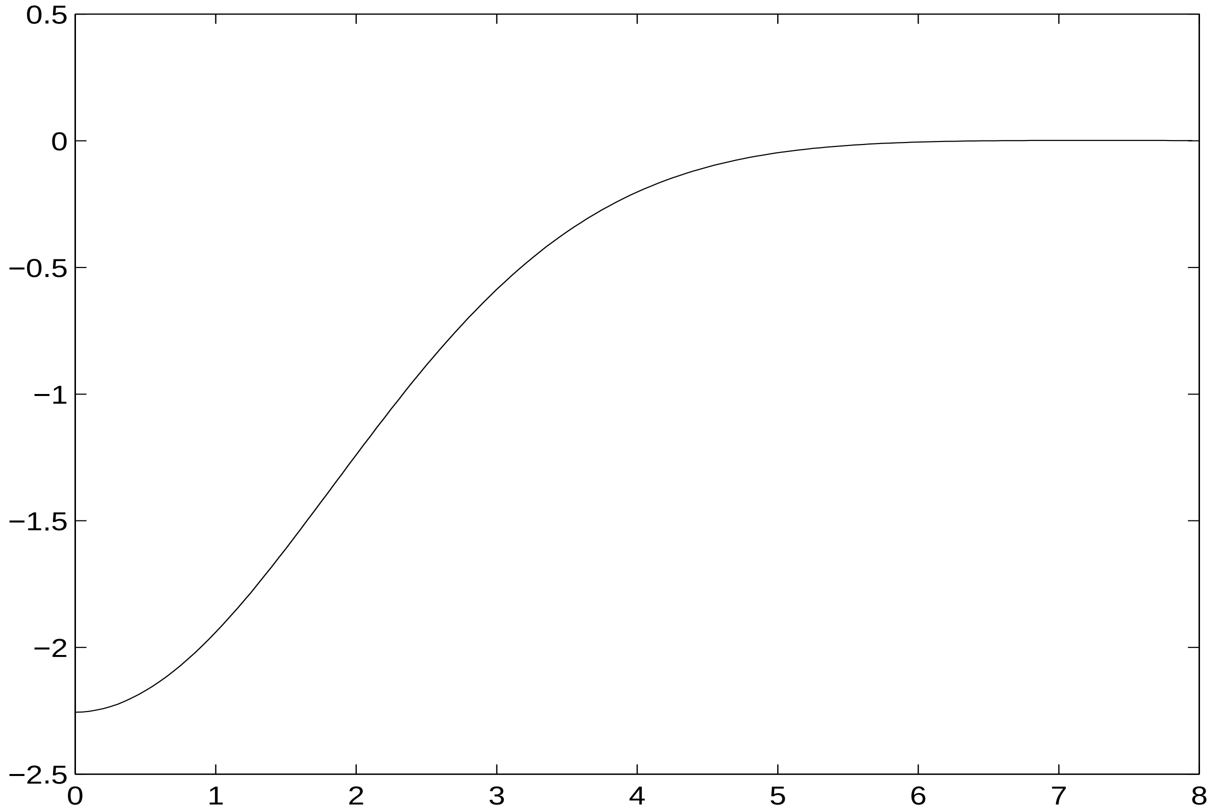

Fig1: The wave function of the D20 brane as a function of the radial coordinate r. The plot was obtained by taking the Fourier transform of the numerical solution of equation (18).

The above form for the wave function is typical. The value of the wave function at the origin in position space decreased from ≈ -0.62 for the D24 brane to ≈ -4.46 for the D18 brane. The point at which the wave function reaches zero is very nearly constant for all the Dp-branes considered here. In figure 2 we have shown the error in the D24 brane solution. The tensions of these solutions was evaluated directly in momentum space. For example, in the case of the D24 brane, we compute

$$\begin{aligned}\hat{T}_{24} = 2\pi^2 T_{25} \quad \int \quad & \frac{dp}{2\pi}\Big(\frac{1}{2}\phi(p)(p^2-1)e^{cp^2}\phi(-p) + \phi(p)\phi(-p) \\ + \quad & \frac{g}{3}\int \frac{dk}{2\pi}\phi(k)\phi(p)\phi(-p-k)\Big)\end{aligned}$$

$$= \frac{2\pi^2 T_{25}}{6} \int \frac{dp}{2\pi}\Big(\phi(p)(p^2-1)e^{cp^2}\phi(-p) + 2\phi(p)\phi(-p)\Big). \qquad (21)$$

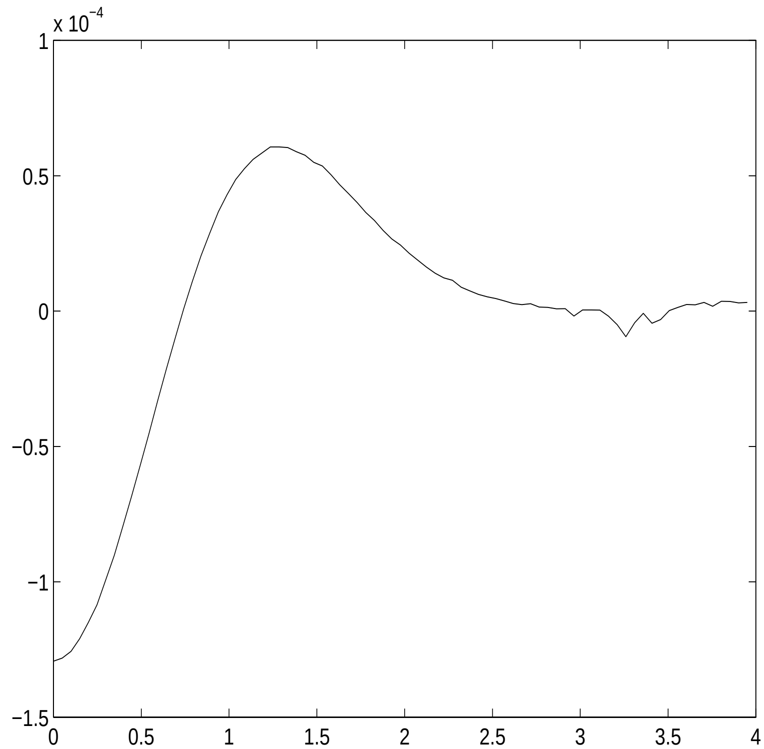

Fig2: The error in the wave function of the D20 brane as a function of the radial coordinate in momentum space p. The plot shows $\mathcal{E}/(\sum_n \hat{G}(k_n)$ as a function of k_n.

In our conventions, a Dp-brane has tension

$$T_p = (2\pi)^{25-p} T_{25}. \qquad (22)$$

In the table below we compare our numerically evaluated tensions, $\hat{T}_p$ with the above values.

p	$\Delta\ T_p$	$\hat{T}_p/T_p$	$\phi(0)$
24	-29.4	0.706	-0.63
23	-27.5	0.725	-0.86
22	-14.3	0.857	-1.19
21	-8.3	0.917	-1.63
20	6.4	1.064	-2.26
19	25.3	1.253	-3.15
18	64.0	1.640	-4.46

Table I

Values of the Dp brane tensions computed using the numerical tachyon lump solution. The parameter ΔT_p is defined as $\Delta T_p = \frac{\hat{T}_p - T_p}{T_p} \times 100$. $\phi(0)$ is the value of the wavefunction at the origin in position space.

Clearly there is no obstacle for the existence of lower p-Brane solutions. The trend is that the static lump gives systematicaly a growing tension. This is actually opposite to what one finds in the extreme field theory limit. Concerning further improvement of present results one expects (esspecialy for lower p) the relevance of higher massive levels. It is next important to study the direction of their contributions. It is also relevant to perform a similar study in superstring theory[1, 6, 7, 16].

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